Risk Dynamics in the Eurozone: A New Factor Model for Sovereign CDS and Equity Returns

Petros Dellaportas[a], Loukia Meligkotsidou[b], Roberto Savona[c,*] and Ioannis D. Vrontos[a]

[a]Department of Statistics, Athens University of Economics and Business, Athens, Greece

[b]Department of Mathematics, University of Athens, Athens, Greece

[c]Department of Business Studies, Faculty of Economics, University of Brescia, Brescia, Italy

Abstract

Extending previous work on factor models, this paper introduces the idea of modelling financial returns using a new dynamic factor model. The proposed dynamic factor model provides a way of understanding how financial returns are affected by latent, sector-based factors and macro-systemic risk factors. We propose a Bayesian approach to inference based on Markov chain Monte Carlo (MCMC) methods exploring the risk dynamics in the Eurozone analyzing 17 sovereign CDS and 545 equity returns over the period 2/1/2007-30/8/2013.

Keywords: Factor models; Bayesian methods; Risk factors.

*Corresponding author. Department of Business Studies, Faculty of Economics, University of Brescia, Brescia, Italy. Tel: +39 0302988557, Fax:+39 030295814. Email: savona@eco.unibs.it

1 Introduction

In this paper we introduce a novel Bayesian factor model based on Markov chain Monte Carlo (MCMC) methods to inspect how risks in the financial system are interconnected within the Eurozone. The proposed dynamic factor model explains how financial returns are affected by latent, sector-based factors and macro-systemic risk factors. We explore the risk dynamics in the Eurozone by analyzing 17 sovereign CDS and 545 equity returns of the Eurostoxx 600 index over the period 2/1/2007-30/8/2013.

The remainder of the paper is organized as follows. The proposed dynamic factor model is introduced in section 2. A Bayesian approach to inference based on MCMC methods is presented in section 3. A simulation study is presented in section 4 in order to assess the performance of the inferential method. In section 5 we describe the empirical analysis and illustrate the above model and methods. Finally, we conclude in section 6 with a brief discussion.

2 The Dynamic factor model

In this section we present the dynamic factor model we use to analyse financial return series. In our analysis it is of great interest to identify the risk factors which are most relevant to explain financial returns. We consider $J = 3$ different sectors (Sovereign, Banks and Financial Intermediaries, Corporations), each one composed by $m_j, j = 1, ..., J$, financial assets (CDS and Equities). For these assets we consider the corresponding returns, $r^j_{i,t}, i = 1, ..., m_j, j = 1, ..., J$, in every time point, $t, t = 1, ..., T$. We assume that the return $r^j_{i,t}$ is linked to a set of p_j local covariates, denoted by $Z^j_{i,t}$, which are asset-specific, and a sector systemic risk factor v^j_t, common accoss the assets of the jth sector, as follows:

$$r^j_{i,t} = Z^{j\prime}_{i,t}\alpha^j_i + \beta^j_{i,t}v^j_t + \varepsilon^j_{i,t},\ \varepsilon^j_{i,t}\tilde{}N\left(0,\sigma^2_{ij}\right),\ t = 1, ..., T, i = 1, ..., m_j,\ j = 1, ..., J, \tag{1}$$

where the factor loadings, $\beta^j_{i,t}$, are assumed to be time-varying and follow a pseudo-stochastic mean reverting process, in which a set of q_j structural sector-based covariates, G^j_t, are mixed together with a stochastic component $\mu^j_{i,t}$ through the following equation:

$$\beta^j_{i,t} = \beta^j_{ic} + \varphi^j_i\left(\beta^j_{i,t-1} - \beta^j_{ic}\right) + G^{j'}_t A^j_i + \mu^j_{i,t},\ \mu^j_{i,t}\tilde{}N\left(0,\psi^2_{ij}\right),\ t = 1, ..., T, i = 1, ..., m_j,\ j = 1, ..., J. \tag{2}$$

For identifiability reasons, the factor loadings are assumed to be positive, with $\beta^j_{i,0} = 0, i = 1, ..., m_j, j = 1, ..., J$.

Finally, we assume that all the sector-specific systemic risk factors, v^j_t, are correlated to a macro-systemic risk factor, V_t. Such a macro-factor is in turn related to a set of covariates X_t:

$$v^j_t = \gamma^j V_t + \omega^j_t,\ \omega^j_t\tilde{}N\left(0,k^2_{1j}\right),\ j = 1, ..., J, \tag{3}$$

and

$$V_t = X^{'}_t B + u_t,\ u_t\tilde{}N\left(0,k^2_2\right). \tag{4}$$

The error term variances of the latent factors $v^j_t, j = 1, ..., J$, and V_t are set equal to 1 for identifiability reasons.

3 Estimation and Inference

3.1 The Likelihood

In this section, we present the inferential method adopted to estimate the latent factors and the parameters of the proposed model. The likelihood for the dynamic factor model (1-4) can be written as:

$$
\begin{aligned}
Lik &= \prod_{j=1}^{J}\prod_{t=1}^{T}\prod_{i=1}^{m_j}\left[\left(\sigma_{ij}^{-2}\right)^{1/2}\exp\left\{-\frac{1}{2}\sigma_{ij}^{-2}\left(r_{i,t}^{j}-Z_{i,t}^{j\prime}\alpha_i^j-\beta_{i,t}^{j}v_t^j\right)^2\right\}\right]\times \\
&\quad \prod_{j=1}^{J}\prod_{t=1}^{T}\prod_{i=1}^{m_j}\left[\left(\psi_{ij}^{-2}\right)^{1/2}\exp\left\{-\frac{1}{2}\psi_{ij}^{-2}\left(\beta_{i,t}^{j}-\beta_{ic}^{j}-\varphi_i^j\left(\beta_{i,t-1}^{j}-\beta_{ic}^{j}\right)-G_t^{j\prime}A_i^j\right)^2\right\}\right]\times \\
&\quad \prod_{j=1}^{J}\prod_{t=1}^{T}\left[\exp\left\{-\frac{1}{2}\left(v_t^j-\gamma^j V_t\right)^2\right\}\right]\times\prod_{t=1}^{T}\left[\exp\left\{-\frac{1}{2}\left(V_t-X_t^{\prime}B\right)^2\right\}\right].
\end{aligned}
$$

Then, assuming a prior distribution for the model parameters, $P\left(\alpha,\sigma^2,\beta_c,\varphi,A,\psi^2,\gamma,B\right)$, the joint posterior distribution for all the unknown quantities in our latent factor model can be written as:

$$
\begin{aligned}
& P\left(\alpha,\beta,v,\sigma^2,\beta_c,\varphi,A,\psi^2,\gamma,V,B|r\right) \\
\propto\ & P\left(r|\alpha,\beta,v,\sigma^2,\beta_c,\varphi,A,\psi^2,\gamma,V,B\right)\times P\left(\alpha,\beta,v,\sigma^2,\beta_c,\varphi,A,\psi^2,\gamma,V,B\right) \\
=\ & P\left(r|a,\beta,v,\sigma^2\right)\times P\left(\beta|\beta_c,\varphi,A,\psi^2\right)\times P\left(v|\gamma,V\right)\times P\left(V|B\right)\times P\left(\alpha,\sigma^2,\beta_c,\varphi,A,\psi^2,\gamma,B\right).
\end{aligned}
$$

where α denotes the parameter set consisting of all $\alpha_i^j, i=1,...,m_j, j=1,...,J$, and the respective notation is used for all parameter sets.

3.2 Prior Specification

In our analysis we consider prior independence among the model parameters and use the following prior specification:

$$P\left(\alpha_i^j\right) \propto \exp\left\{-\frac{1}{2}\left(a_i^j-\xi_1^j\right)^{\prime}\Omega_1^{j^{-1}}\left(A_i^j-\xi_1^j\right)\right\}\equiv N_{p_j}\left(\xi_1^j,\Omega_1^j\right),\ i=1,...,m_j,\ j=1,...,J$$

$$P\left(A_i^j\right) \propto \exp\left\{-\frac{1}{2}\left(A_i^j-\xi_2^j\right)^{\prime}\Omega_2^{j^{-1}}\left(A_i^j-\xi_2^j\right)\right\}\equiv N_{q_j}\left(\xi_2^j,\Omega_2^j\right),i=1,...,m_j,\ j=1,...,J$$

$$P\left(B\right) \propto \exp\left\{-\frac{1}{2}\left(B-\xi_0\right)^{\prime}\Omega_0^{-1}\left(B-\xi_0\right)\right\}\equiv N_{q_0}\left(\xi_0,\Omega_0\right)$$

$$P\left(\gamma^j\right) \propto \exp\left\{-\frac{1}{2\sigma_\gamma^2}\left(\gamma^j-\mu_\gamma\right)^2\right\}\equiv N\left(\mu_\gamma,\sigma_\gamma^2\right),\ j=1,...,J$$

$$P\left(\beta_{ic}^j\right) \propto \exp\left\{-\frac{1}{2\sigma_0^2}\left(\beta_{ic}^j-\mu_0\right)^2\right\}\equiv N\left(\mu_0,\sigma_0^2\right),\ i=1,...,m_j,\ j=1,...,J$$

$$P\left(\varphi_i^j\right) \propto \frac{1}{2B\left(c_0,d_0\right)}\left(\frac{1+\varphi_i^j}{2}\right)^{c_0-1}\left(\frac{1-\varphi_i^j}{2}\right)^{d_0-1} \equiv Beta\left(c_0,d_0\right),\ i=1,...,m_j,\ j=1,...,J$$

$$P\left(\sigma_{ij}^{-2}\right) \propto \left(\sigma_{ij}^{-2}\right)^{c_1-1}\exp\left\{-d_1\sigma_{ij}^{-2}\right\} \equiv Ga\left(c_1,d_1\right),\ i=1,...,m_j,\ j=1,...,J$$

$$P\left(\psi_{ij}^{-2}\right) \propto \left(\psi_{ij}^{-2}\right)^{c_2-1}\exp\left\{-d_2\psi_{ij}^{-2}\right\} \equiv Ga\left(c_2,d_2\right),\ i=1,...,m_j,\ j=1,...,J.$$

3.3 Full Conditional Posterior Distributions

Next we compute the full conditional posterior distributions for the parameters of our dynamic factor model. The derived full conditional distributions for the precision (inverted variance) parameters are given by:

$$\begin{aligned} P\left(\sigma_{ij}^{-2}|\cdot\right) &\propto \left(\sigma_{ij}^{-2}\right)^{T/2}\exp\left\{-\frac{1}{2}\sigma_{ij}^{-2}\sum_{t=1}^{T}\left(r_{i,t}^j - Z_{i,t}^{j\prime}\alpha_i^j - \beta_{i,t}^j v_t^j\right)^2\right\}\times\left(\sigma_{ij}^{-2}\right)^{c_1-1}\exp\left\{-d_1\sigma_{ij}^{-2}\right\} \\ &\equiv Ga\left(\frac{T}{2}+c_1, \frac{1}{2}\sum_{t=1}^{T}\left(r_{i,t}^j - Z_{i,t}^{j\prime}\alpha_i^j - \beta_{i,t}^j v_t^j\right)^2 + d_1\right),\ i=1,...,m_j,\ j=1,...,J \end{aligned}$$

$$\begin{aligned} P\left(\psi_{ij}^{-2}|\cdot\right) &\propto \left(\psi_{ij}^{-2}\right)^{T/2}\exp\left\{-\frac{1}{2}\psi_{ij}^{-2}\sum_{t=1}^{T}\left(\beta_{i,t}^j - \beta_{ic}^j - \varphi_i^j\left(\beta_{i,t-1}^j - \beta_{ic}^j\right) - G_t^{j'}A_i^j\right)^2\right\}\times\left(\psi_{ij}^{-2}\right)^{c_2-1}\exp\left\{-d_2\psi_{ij}^{-2}\right\} \\ &\equiv Ga\left(\frac{T}{2}+c_2, \frac{1}{2}\sum_{t=1}^{T}\left(\beta_{i,t}^j - \beta_{ic}^j - \varphi_i^j\left(\beta_{i,t-1}^j - \beta_{ic}^j\right) - G_t^{j'}A_i^j\right)^2 + d_2\right),\ i=1,...,m_j,\ j=1,...,J \end{aligned}$$

The full conditional posterior distributions for the latent factors v_t^j and the parameters α_i^j of equation (1) are given by:

$$\begin{aligned} P\left(v_t^j|\cdot\right) &\propto \prod_{i=1}^{m_j}\left[\exp\left\{-\frac{1}{2}\sigma_{ij}^{-2}\left(r_{i,t}^j - Z_{i,t}^{j\prime}\alpha_i^j - \beta_{i,t}^j v_t^j\right)^2\right\}\right]\times\exp\left\{-\frac{1}{2}\left(v_t^j - \gamma^j V_t\right)^2\right\} \\ &\equiv N\left(\frac{\gamma^j V_t + \sum_{i=1}^{m_j}\sigma_{ij}^{-2}\beta_{i,t}^j\left(Z_{i,t}^{j\prime}r_{i,t}^j - \alpha_i^j\right)}{1+\sum_{i=1}^{m_j}\sigma_{ij}^{-2}\left(\beta_{i,t}^j\right)^2}, \frac{1}{1+\sum_{i=1}^{m_j}\sigma_{ij}^{-2}\left(\beta_{i,t}^j\right)^2}\right),\ j=1,...,J,\ t=1,...,T \end{aligned}$$

$$\begin{aligned} P\left(\alpha_i^j|\cdot\right) &\propto \exp\left\{-\frac{1}{2}\sigma_{ij}^{-2}\sum_{t=1}^{T}\left(r_{i,t}^j - Z_{i,t}^{j\prime}\alpha_i^j - \beta_{i,t}^j v_t^j\right)^2\right\}\times\exp\left\{-\frac{1}{2}\left(a_i^j - \xi_1^j\right)'\Omega_1^{j^{-1}}\left(A_i^j - \xi_1^j\right)\right\} \\ &\equiv N_{p_j}\left(\xi_{1*}^j, \Omega_{1*}^j\right),\ i=1,...,m_j,\ j=1,...,J, \\ \text{where } \xi_{1*}^j &= \Omega_{1*}^j\left[\Omega_1^{j^{-1}}\xi_1^j + \sigma_{ij}^{-2}Z_{i,t}^{j\prime}\left(r_i^j - diag(v^j)\beta_i^j\right)\right],\text{ and } \Omega_{1*}^j = \left[\Omega_1^{j^{-1}} + \sigma_{ij}^{-2}Z_i^{j\prime}Z_i^j\right]^{-1}. \end{aligned}$$

where r_i^j and Z_i^j are the vector of returns and the local covariates design matrix for the ith asset of the jth sector, and $diag\left(v^j\right)$ is a $T\times T$ diagonal matrix with elements v_1^j, v_2^j,..., v_T^j in the main diagonal.

The full conditional posterior distributions for the parameters β^j_{ic}, φ^j_i, and A^j_i of equation (2) are given by:

$$
\begin{aligned}
P\left(\beta^j_{ic}|\cdot\right) &\propto \exp\left\{-\frac{1}{2}\psi^{-2}_{ij}\sum_{t=1}^{T}\left(\beta^j_{i,t}-\varphi^j_i\beta^j_{i,t-1}-G^{j'}_t A^j_i-\left(1-\varphi^j_i\right)\beta^j_{ic}\right)^2\right\}\times\exp\left\{-\frac{1}{2\sigma_0^2}\left(\beta^j_{ic}-\mu_0\right)^2\right\} \\
&\equiv N\left(\mu^*_0,(\sigma^*_0)^2\right),\ i=1,...,m_j,\ j=1,...,J, \\
\text{where } \mu^*_0 &= (\sigma^*_0)^2\left[\sigma_0^{-2}\mu_0+\psi^{-2}_{ij}\left(1-\varphi^j_i\right)\sum_{t=1}^{T}\left(\beta^j_{i,t}-\varphi^j_i\beta^j_{i,t-1}-G^{j'}_t A^j_i\right)\right], \\
\text{and } (\sigma^*_0)^2 &= \left[T\psi^{-2}_{ij}\left(1-\varphi^j_i\right)^2+\sigma_0^{-2}\right]^{-1}
\end{aligned}
$$

$$
P\left(\varphi^j_i|\cdot\right)\propto\exp\left\{-\frac{1}{2}\psi^{-2}_{ij}\sum_{t=1}^{T}\left(\beta^j_{i,t}-\beta^j_{ic}-\varphi^j_i\left(\beta^j_{i,t-1}-\beta^j_{ic}\right)-G^{j'}_t A^j_i\right)^2\right\}\times\left(\frac{1+\varphi^j_i}{2}\right)^{c_0-1}\left(\frac{1-\varphi^j_i}{2}\right)^{d_0-1}
$$

$$
\begin{aligned}
P\left(A^j_i|\cdot\right) &\propto \exp\left\{-\frac{1}{2}\psi^{-2}_{ij}\sum_{t=1}^{T}\left(\beta^j_{i,t}-\beta^j_{ic}-\varphi^j_i\left(\beta^j_{i,t-1}-\beta^j_{ic}\right)-G^{j'}_t A^j_i\right)^2\right\}\times\exp\left\{-\frac{1}{2}\left(A^j_i-\xi^j_2\right)'\Omega^{j^{-1}}_2\left(A^j_i-\xi^j_2\right)\right\} \\
&\equiv N_{q_i}\left(\xi^j_{2*},\Omega^j_{2*}\right),\ i=1,...,m_j,\ j=1,...,J, \\
\text{where } \xi^j_{2*} &= \Omega^j_{2*}\left[\Omega^{j^{-1}}_2\xi^j_2+\psi^{-2}_{ij}G^{j'}\left(\beta^j_i-\beta^j_{ic}-\varphi^j_i\left(\beta^j_{i,-1}-\beta^j_{ic}\right)\right)\right],\text{ and } \Omega^j_{2*}=\left[\Omega^{j^{-1}}_2+\psi^{-2}_{ij}G^{j'}G^j\right]^{-1}.
\end{aligned}
$$

The full conditional posterior distributions for the time-varying parameters $\beta^j_{i,t}$, $t=1,...,T$ of equation (2) are given below. In order to draw the vector $\beta^j_i=\left(\beta^j_{i,1},...,\beta^j_{i,T}\right)'$ jointly, given everything else, from its full conditional posterior, it is convenient to write equations (1-2) in the form:

$$
r^j_i=Z^j_i\alpha^j_i+diag\left(v^j\right)\beta^j_i+\varepsilon^j_i,\ \varepsilon^j_i\tilde{}N_T\left(0_T,\sigma^2_{ij}I_T\right),\ i=1,...,m_j,\ j=1,...,J,
$$

where r^j_i is a $T\times 1$ vector, Z^j_i is a $T\times p_j$ design matrix , $diag\left(v^j\right)$ is a $T\times T$ diagonal matrix with elements v^j_1, v^j_2,..., v^j_T in the diagonal, ε^j_i is a $T\times 1$ vector of innovations, I_T is the $T\times T$ identity matrix. In a similar form equation (2) can be written as:

$$
\beta^j_i=\beta^j_{ic}1_T+\varphi^j_i\left(S_1\beta^j_i-\beta^j_{ic}1_T\right)+G^jA^j_i+\mu^j_i,\ \mu^j_i\tilde{}N_T\left(0_T,\psi^2_{ij}I_T\right),\ i=1,...,m_j,\ j=1,...,J,
$$

where 1_T is a $T\times 1$ vector of ones and S_1 is a $T\times T$ matrix of the form $S_1=\begin{bmatrix}0&0&\dots&0&0\\1&0&\dots&0&0\\0&1&\dots&0&0\\\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&\dots&1&0\end{bmatrix}$.

Then, the full conditional posterior distribution of the vector β_i^j can be computed as:

$$\begin{aligned}
P\left(\beta_i^j|\cdot\right) &\propto \exp\left\{-\frac{1}{2}\sigma_{ij}^{-2}\left(r_i^j - Z_i^j\alpha_i^j - diag\left(v^j\right)\beta_i^j\right)^{'}\left(r_i^j - Z_i^j\alpha_i^j - diag\left(v^j\right)\beta_i^j\right)\right\}\times \\
&\quad \exp\left\{\begin{array}{c} -\frac{1}{2}\psi_{ij}^{-2}\left(\beta_i^j - \beta_{ic}^j 1_T - \varphi_i^j S_1\beta_i^j + \varphi_i^j\beta_{ic}^j 1_T - G^j A_i^j\right)\times \\ \times\left(\beta_i^j - \beta_{ic}^j 1_T - \varphi_i^j S_1\beta_i^j + \varphi_i^j\beta_{ic}^j 1_T - G^j A_i^j\right)\end{array}\right\} Ind\{\beta_i^j > 0\} \\
&= N_T\left(\xi_{ij}^*, \Omega_{ij}^*\right) Ind\{\beta_i^j > 0\},\ i = 1, ..., m_j,\ j = 1, ..., J, \\
\text{where } \xi_{ij}^* &= \Omega_{ij}^*\left[\sigma_{ij}^{-2} diag\left(v^j\right)\left(r_i^j - Z_i^j\alpha_i^j\right) + \psi_{ij}^{-2}\left(I_T - \varphi_i^j S_1\right)^{'}\left(\beta_{ic}^j 1_T - \varphi_i^j\beta_{ic}^j 1_T + G^j A_i^j\right)\right], \\
\text{and } \Omega_{ij}^* &= \left[\sigma_{ij}^{-2} diag\left(v^j\right) diag\left(v^j\right) + \psi_{ij}^{-2}\left(I_T - \varphi_i^j S_1\right)^{'}\left(I_T - \varphi_i^j S_1\right)\right]^{-1},
\end{aligned}$$

where $Ind\{\beta_i^j > 0\}$ is an indicator function which satisfies the constraint that all facttor loading should be positive.

The full conditional posterior distributions for the parameters γ^j, and the latent factors V_t of equation (3) are given by:

$$\begin{aligned}
P\left(\gamma^j|\cdot\right) &\propto \exp\left\{-\frac{1}{2}\sum_{t=1}^{T}\left(v_t^j - \gamma_j V_t\right)^2\right\}\times\exp\left\{-\frac{1}{2\sigma_\gamma^2}\left(\gamma^j - \mu_\gamma\right)^2\right\} \\
&= N\left(\frac{\sigma_\gamma^{-2}\mu_\gamma + \sum\limits_{t=1}^{T} v_t^j V_t}{\sigma_\gamma^{-2} + \sum\limits_{t=1}^{T} V_t^2}, \frac{1}{\sigma_\gamma^{-2} + \sum\limits_{t=1}^{T} V_t^2}\right).
\end{aligned}$$

$$\begin{aligned}
P\left(V_t|\cdot\right) &\propto \prod_{j=1}^{J}\exp\left\{-\frac{1}{2}\left(v_t^j - \gamma^j V_t\right)^2\right\}\times\exp\left\{-\frac{1}{2}\left(V_t - X_t^{'}B\right)^2\right\} \\
&= N\left(\frac{X_t^{'}B + \sum\limits_{j=1}^{J}\gamma^j v_t^j}{1 + \sum\limits_{j=1}^{J}\gamma^{j2}}, \frac{1}{1 + \sum\limits_{j=1}^{J}\gamma^{j2}}\right),\ t = 1, ..., T.
\end{aligned}$$

The full conditional posterior distributions for parameters B of equation (4) are given by:

$$\begin{aligned}
P\left(B|\cdot\right) &\propto \exp\left\{-\frac{1}{2}\left(V - XB\right)^{'}\left(V - XB\right)\right\}\times\exp\left\{-\frac{1}{2}\left(B - \xi_0\right)^{'}\Omega_0^{-1}\left(B - \xi_0\right)\right\} \\
&\equiv N_{q_0}\left(\xi_0^*, \Omega_0^*\right) \\
\text{where } \xi_0^* &= \Omega_0^*\left[X^{'}V + \Omega_0^{-1}\xi_0\right], \text{ and } \Omega_0^* = \left[X^{'}X + \Omega_0^{-1}\right]^{-1}.
\end{aligned}$$

4 Simulation Study

In this section, we conduct several simulation experiments in order to assess the inferential method based on the proposed MCMC algorithm. The aim of this study is to assess the performance of the Bayesian methodology, to estimate the model parameters, the latent factors, the time-varying betas as well as the macro-systemic risk factor. We conduct a series of simulation experiments considering different factor models starting from the simple factor model of Lopez and West (2004). We consider different sample sizes of time series, i.e. $T = 100$, $T = 200$ and $T = 500$ data points, however we present the simulation experiments only for $T = 100$ for reasons of space. First, we simulate data from the simple factor model M_1:

$$r_{i,t}^j = \alpha_i^j + \beta_i^j v_t^j + \varepsilon_{i,t}^j,\ \varepsilon_{i,t}^j \tilde{} N\left(0, \sigma_{ij}^2\right),\ i = 1, ..., m_j,\ j = 1, ..., J,$$

where the factors v_t^j follow a standard normal distribution, i.e.

$$v_t^j \tilde{} N\left(0, 1\right), j = 1, ..., J.$$

In our simulation experiments we have used $J = 3$ sectors, with $m_1 = 2$, $m_2 = 3$ and $m_3 = 5$, that is there are two assets in the first sector, three assets in the second and five assets in the third sector. We run the MCMC algorithm for $10,000$ iterations using a burn-in period of $1,000$ iterations and estimate the model parameters and the latent factors. Figure 1 illustrates the posterior means for the latent factors v_t^j, $j = 1, 2, 3$ (red line) and the true factor values (blue line) simulated based on model specification M_1. This figure indicates the accuracy of the estimates of the latent factors to the true ones. We also evaluate the performance of the latent factor estimates by using the following statistic (Doz, Giannone and Reichlin, 2006, Korobilis and Schumacher, 2014):

$$SSF0 = \frac{tr\left[v'\widehat{v}\left(\widehat{v}'\widehat{v}\right)^{-1}\widehat{v}'v\right]}{tr\left[v'v\right]},$$

where $\widehat{v}$ denotes the latent factor estimates and v denotes the true simulated factor. This statistic takes values between zero and one, and therefore, values of $SSF0$ close to one indicate a very good approximation of the true latent factors. Based on the results presented in Table 1 (line 1 for model M_1) we observe that the SSF0 statistics ranges from 0.922 to 0.944, indicating a very good approximation of the estimated latent factors to the true simulated ones.

Insert Table 1 about here

Insert Figure 1 about here

Next, we simulate data from a factor model, M_2 that allows for time-varying betas:

$$r_{i,t}^j = \alpha_i^j + \beta_{i,t}^j v_t^j + \varepsilon_{i,t}^j,\ \varepsilon_{i,t}^j \tilde{} N\left(0, \sigma_{ij}^2\right),\ i = 1, ..., m_j,\ j = 1, ..., J,$$

where the parameters $\beta_{i,t}^j$ are time-varying and assume a Random Walk process through the following equation:

$$\beta_{i,t}^j = \beta_{i,t-1}^j + \mu_{i,t}^j,\ \mu_{i,t}^j \tilde{} N\left(0, \psi_{ij}^2\right),\ i = 1, ..., m_j,\ j = 1, ..., J,\ t = 1, ...T$$

with $\beta_{i,0}^j = 0$ and $\beta_{i,t}^j > 0$, i.e. set the initial condition of the factor loadings equal to zero, and impose the factor loadings during the time periods to be positive in order to identify the model (see, for example, Geweke and Zhou, 1996, Lopez and West, 2004). The factors, v_t^j, follow a standard normal distribution i.e.

$$v_t^j \tilde{} N\left(0, 1\right),\ j = 1, ..., J.$$

Table 1 (second line for model M_2) and Table 2 (first line for model M_2) present the performance evaluation statistic SSF0 for the estimated latent factors v_t^j and for the estimated time-varying $\beta_{i,t}^j$ parameters, respectively. The values of this statistic are very high, near to one, indicating a very good approximation of the estimated latent factors and beta parameters to the corresponding true values used in the simulation. This is also depicted in Figure 2, which illustrate the posterior means for the estimated latent factors v_t^j, $j = 1, 2, 3$ (red line) and the true factor values (blue line), and in Figure 3, which presents the posterior means for the estimated time-varying parameters $\beta_{i,t}^j$, $j = 1, 2, 3$ (red line) and true time-varying beta values (blue line) simulated based on model specification M_2.

Insert Table 2 about here

Insert Figure 2 - Figure 3 about here

Finally, we simulate data from the following dynamic factor model, M_3:

$$r_{i,t}^j = \alpha_i^j + \beta_{i,t}^j v_t^j + \varepsilon_{i,t}^j,\ \varepsilon_{i,t}^j \tilde{}\, N\left(0, \sigma_{ij}^2\right),\ i = 1, ..., m_j,\ j = 1, ..., J,$$

where the parameters $\beta_{i,t}^j$ are time-varying and assume a Random Walk process through the following equation

$$\beta_{i,t}^j = \beta_{i,t-1}^j + \mu_{i,t}^j,\ \mu_{i,t}^j \tilde{}\, N\left(0, \psi_{ij}^2\right),\ i = 1, ..., m_j,\ j = 1, ..., J.$$

We assume that each sector j has a specific systemic risk factor v_t^j correlated to a macro-systemic risk factor V_t. Such a macro-factor is in turns related to a set of covariates X_t:

$$v_t^j = \alpha_1^j V_t + \omega_t^j,\ \omega_t^j \tilde{}\, N\left(0, k_{1j}^2 = 1\right),\ j = 1, ..., J,$$

and

$$V_t = X_t^{'} B + u_t,\ u_t \tilde{}\, N\left(0, k_2^2 = 1\right).$$

Table 1 (third line for model M_3) and Table 2 (second line for model M_3) present the performance evaluation statistic SSF0 for the estimated latent factors v_t^j, the estimated macro-systemic risk factor V_t, and for the estimated time-varying $\beta_{i,t}^j$ parameters, respectively. And in this simulation experiment, the statistic SSF0 receives high values, indicating a very good approximation of the estimated quantities to the corresponding true ones. This is also depicted in Figures 4-6, which illustrate the posterior means for the estimated quantities versus the true values used in the simulation experiment based on model specification M_3.

Insert Figure 4 - Figure 6 about here

Based on the result of the simulation experiments we can conclude that the proposed Bayesian inferential procedure is able to estimate accurately the unknown factors and time-varying parameters of the dynamic factor model.

5 Empirical Application

In our empirical analysis we used daily returns for 17 sovereign CDS of the Eurozone and 545 equity returns of the Eurostoxx 600 index over the period 2/1/2007-30/8/2013. Sovereign CDS are used to measure sovereign risks, while equities are used to estimate risk dynamics for financials (what we mean as Bank and other Financial Intermediaries) and non-financials (Corporations). As we described in section 2, our system of equations includes sector-based and common covariates depending on the financial asset return we are modelling (sovereign CDS, equity financial, equity non-financial). Specifically, we used:

Euro Sovereign CDSs

- Country-specific covariates (equation 1): (1) real GDP growth; (2) export/GDP; (3) unemployment rate; (4) M3/GDP; (5) Debt/GDP; (6); domestic industrial production; (7) domestic inflation; (8) domestic equity index returns.

- Sector-specific covariates (equation 2): (9) Volatility premium (VIX minus the realized volatility over the next 30 days); (10) Liquidity spread (Euribor 3m minus Eonia 3m); (11) Euro/US Dollar exchange rate variations.

Euro Stocks

- Country-specific covariates (equation 1): (1) sectorial index (Financial vs. Non Financial); (2) domestic industrial production; (3) domestic inflation.

- Sector-specific covariates (equation 2): (4) Momentum (6m minus 1m Eurostoxx 600 computed at time t by $P_{t-21}/P_{t-126}-1$ where P_t is the asset price at time t, in order to avoid the 1-month reversal period); (5) Risk Premium Europe (Stoxx Europe 600 earning per share minus (0.70 $\times$ [BofA ML 7-10y Euro Non-Financial] + 0.30 $\times$ [BofA ML 7-10y Sterling Corporating Non-Financial])); (6) Risk Premium US (S&P 500 earning per share minus BofA ML US Corporate 7-10y Yield).

Macro-systemic risk factor

- systemic covariates (equation 4): (1) Credit Spread (US Corp BBB/Baa minus US Treasury 10 yr); (2) US Tbill 3m; (3) Euro Term Spread (10 yr minus 2 yr government bond yield); (4) VIX.

A first step in the empirical application is to estimate the simple factor model M_1:

$$r_{i,t}^{j} = \alpha_i^j + \beta_i^j v_t^j + \varepsilon_{i,t}^j,\ \varepsilon_{i,t}^j \tilde{} N\left(0, \sigma_{ij}^2\right),\ i = 1, ..., m_j,\ j = 1, ..., J,$$

where the factors v_t^j follow a standard normal distribution, i.e.

$$v_t^j \tilde{} N\left(0, 1\right), j = 1, ..., J.$$

In this application we use $J = 3$ sectors, with $m_1 = 7$, $m_2 = 6$ and $m_3 = 7$, that is there are seven sovereign CDS in the first sector, six assets in the Financial sector and seven assets in the Non-Financial sector. In particular, we use the CDS of Germany, Greece, Spain, France, Ireland, Italy and Portugal, the financial assets are Banca Popolare Di Milano (Italy), Bank of Ireland (Ireland), Deutsche Bank (Germany), Banco Comercial Portugues (Portugal), Banco Popular Espanol (Spain), BNP Paribas (France), while the Non-Financial assets include Fiat (Italy), Hellenic Telecommunication (Greece), C&C Group (Ireland), Bayer (Germany), EDP Energas (Portugal), Endesa (Spain), Air France (France).

Insert Table 3, Table 4 about here

Insert Figure 7 about here

We run the MCMC algorithm for $20,000$ iterations using a burn-in period of $5,000$ iterations and estimate the model parameters and the latent factors. Figure 7 illustrates the posterior means for the estimated latent factors v_t^j, $j = 1, 2, 3$ (red line). We present in Table 3, the estimated β_i^j parameters and their corresponding standard errors, while tin Table 4 we present the correlation among the β_i^j parameters within each sector.

Acknowledgments

The research leading to these results has received funding from the European Union Seventh Framework Programme (FP7-SSH/2007-2013) under grant agreement no 320270 "SYRTO".

References

Doz, C., Giannone, D. and L. Reichlin (2006). A Quasi Maximum Likelihood Approach for Large Approximate Dynamic Factor Models, *European Central Bank*, Working Paper Series, No. 674.

Geweke, J. and G. Zhou (1996). Measuring the Pricing Error of the Arbitrage Pricing Theory, *The Review of Financial Studies*, 9, 557-587.

Korobilis, D. and C. Schumacher (2014). Efficient Estimation and Forecasting in Dynamic Factor Models with Structural Instability, working paper.

Lopes, H.F., and M. West (2004). Bayesian Model Assesment in Factor Analysis, *Statistica Sinica,* 14, 41-67.

Table 1: Performance evaluation of the estimated latent factors v_t^j [for the three model specifications M_1, M_2, M_3] and of the macro-systemic risk factor V_t [for the third model specifications M_3] based on the $SSF0$ ststistic.

$Models$	v_t^1	v_t^2	v_t^3	$v_t^j all$	V_t
M_1	0.922	0.934	0.944	0.934	–
M_2	0.924	0.903	0.959	0.931	–
M_3	0.946	0.971	0.976	0.969	0.967

Table 2: Performance evaluation of the estimated time-varying parameters $\beta_{i,t}^j$ based on the $SSF0$ ststistic for the two model specifications M_2, M_3.

$Models$	$\beta_{1,t}^1$	$\beta_{2,t}^1$	$\beta_{1,t}^2$	$\beta_{2,t}^2$	$\beta_{3,t}^2$	$\beta_{1,t}^3$	$\beta_{2,t}^3$	$\beta_{3,t}^3$	$\beta_{4,t}^3$	$\beta_{5,t}^3$	$\beta_{i,t}^j all$
M_1	–	–	–	–	–	–	–	–	–	–	–
M_2	0.954	0.941	0.965	0.912	0.958	0.973	0.961	0.932	0.975	0.966	0.988
M_3	0.973	0.963	0.987	0.991	0.979	0.996	0.997	0.993	0.996	0.996	0.996

Table 3: The estimated β_i^j parameters and their corresponding standard errors.

CDS	$\beta_{GER,t}^{cds}$	$\beta_{GR,t}^{cds}$	$\beta_{SP,t}^{cds}$	$\beta_{FR,t}^{cds}$	$\beta_{IRE,t}^{cds}$	$\beta_{IT,t}^{cds}$	$\beta_{PO,t}^{cds}$
$Estimates$	0.699	0.522	0.889	0.739	0.314	0.876	0.782
$Std.Error$	(0.022)	(0.023)	(0.019)	(0.021)	(0.025)	(0.020)	(0.021)
Financials	$\beta_{GER,t}^{Fin}$	$\beta_{GR,t}^{Fin}$	$\beta_{SP,t}^{Fin}$	$\beta_{FR,t}^{Fin}$	$\beta_{IRE,t}^{Fin}$	$\beta_{IT,t}^{Fin}$	$\beta_{PO,t}^{Fin}$
$Estimates$	0.861	-	0.755	0.883	0.457	0.642	0.514
$Std.Error$	(0.020)	-	(0.021)	(0.019)	(0.024)	(0.022)	(0.023)
Non-Financial	$\beta_{GER,t}^{NFin}$	$\beta_{GR,t}^{NFin}$	$\beta_{SP,t}^{NFin}$	$\beta_{FR,t}^{NFin}$	$\beta_{IRE,t}^{NFin}$	$\beta_{IT,t}^{NFin}$	$\beta_{PO,t}^{NFin}$
$Estimates$	0.651	0.405	0.586	0.657	0.340	0.712	0.585
$Std.Error$	(0.024)	(0.026)	(0.025)	(0.024)	(0.026)	(0.024)	(0.025)

Table 4: Correlations of the estimated β_i^j parameters within sector.

Panel A: First sector - CDS								
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
Germany	(1)	1.00						
Greece	(2)	0.22	1.00					
Spain	(3)	0.40	0.28	1.00				
France	(4)	0.35	0.23	0.43	1.00			
Ireland	(5)	0.12	0.08	0.17	0.14	1.00		
Italy	(6)	0.41	0.29	0.60	0.44	0.16	1.00	
Portugal	(7)	0.34	0.25	0.50	0.38	0.15	0.49	1.00
Panel B: Second sector - Financials								
		(1)	(2)	(3)	(4)	(5)	(6)	
Italy	(1)	1.00						
Ireland	(2)	0.16	1.00					
Germany	(3)	0.33	0.23	1.00				
Portugal	(4)	0.18	0.12	0.25	1.00			
Spain	(5)	0.30	0.19	0.42	0.23	1.00		
France	(6)	0.34	0.22	0.53	0.26	0.41	1.00	
Panel C: Third sector - Non-Financials								
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
Italy	(1)	1.00						
Greece	(2)	0.12	1.00					
Ireland	(3)	0.10	0.05	1.00				
Germany	(4)	0.18	0.10	0.10	1.00			
Portugal	(5)	0.10	0.11	0.08	0.19	1.00		
Spain	(6)	0.12	0.09	0.09	0.18	0.22	1.00	
France	(7)	0.27	0.11	0.09	0.15	0.10	0.12	1.00

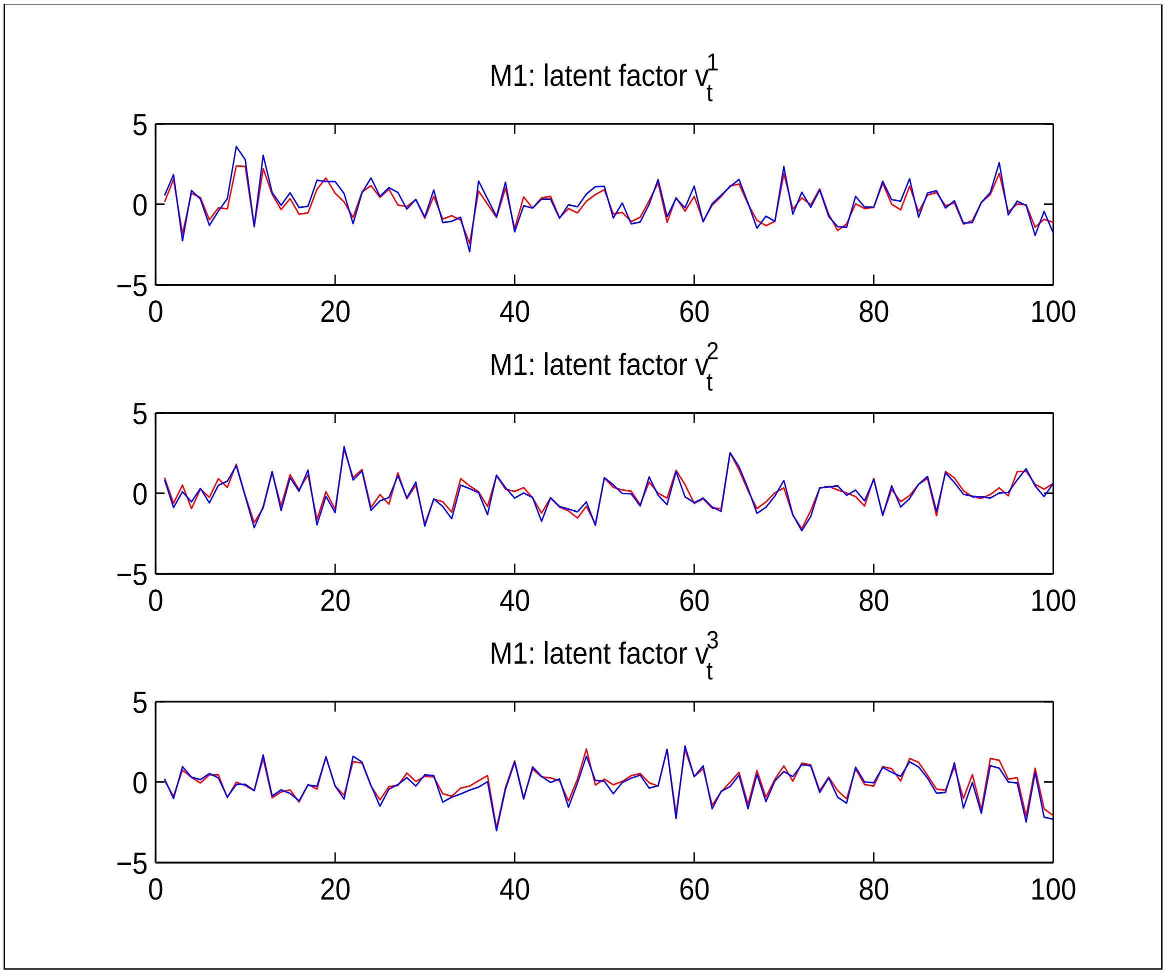

Figure 1: Posterior means for the latent factors v_t^j, $j = 1, 2, 3$, (red line) and true factor values (blue line) simulated based on model specification M_1.

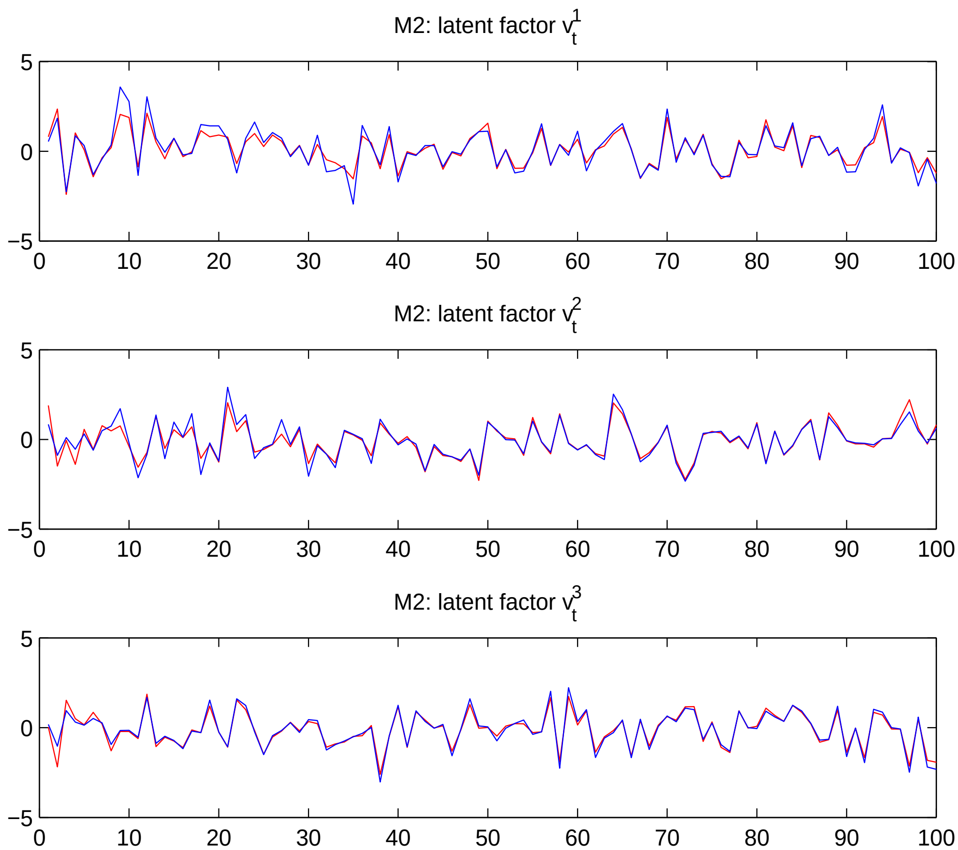

Figure 2: Posterior means for the latent factors v_t^j, $j = 1, 2, 3$, (red line) and true factor values (blue line) simulated based on model specification M_2.

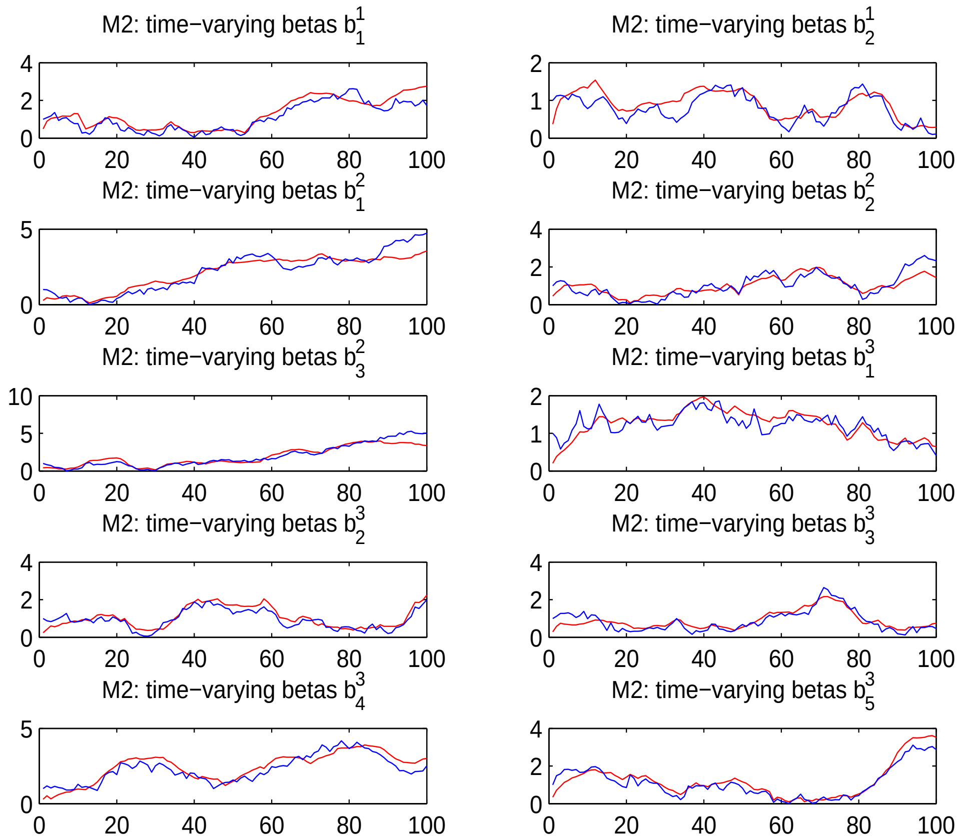

Figure 3: Posterior means for the time-varying parameters $\beta_{i,t}^j$, $j = 1, 2, 3$, (red line) and true time-varying beta values (blue line) simulated based on model specification M_2.

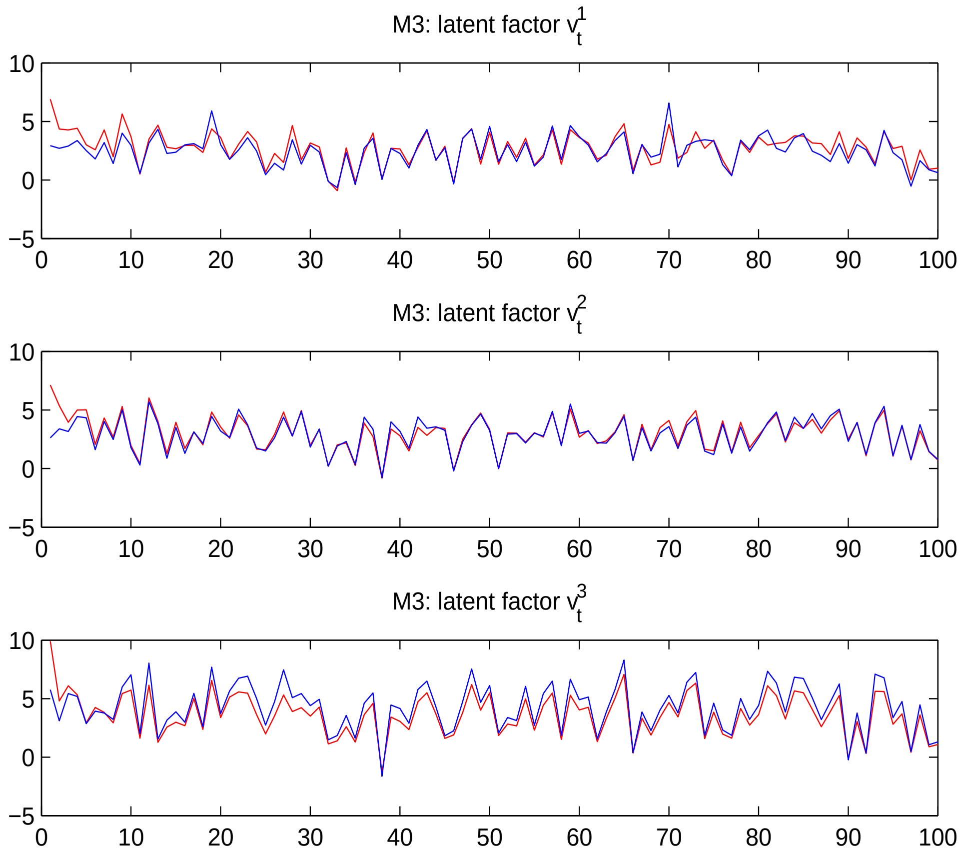

Figure 4: Posterior means for the latent factors v_t^j, $j = 1, 2, 3$, (red line) and true factor values (blue line) simulated based on model specification M_3.

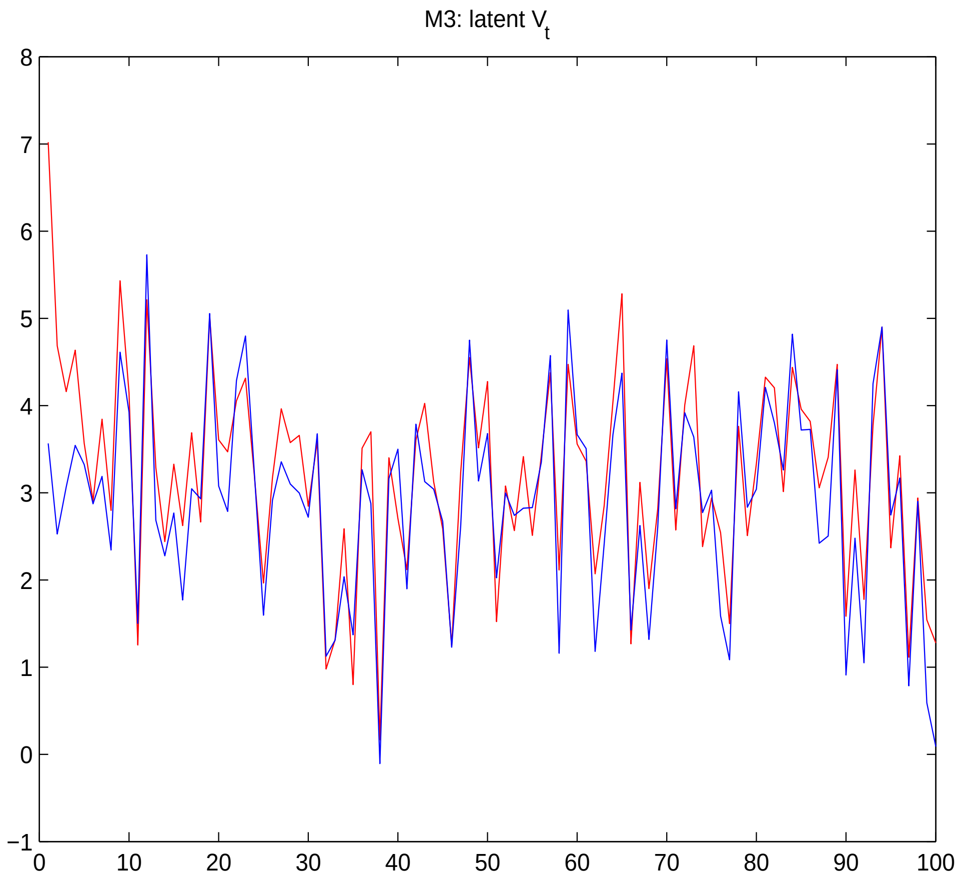

Figure 5: Posterior means for the macro-systemic risk factor V_t (red line) and true risk factor values (blue line) simulated based on model specification M_3.

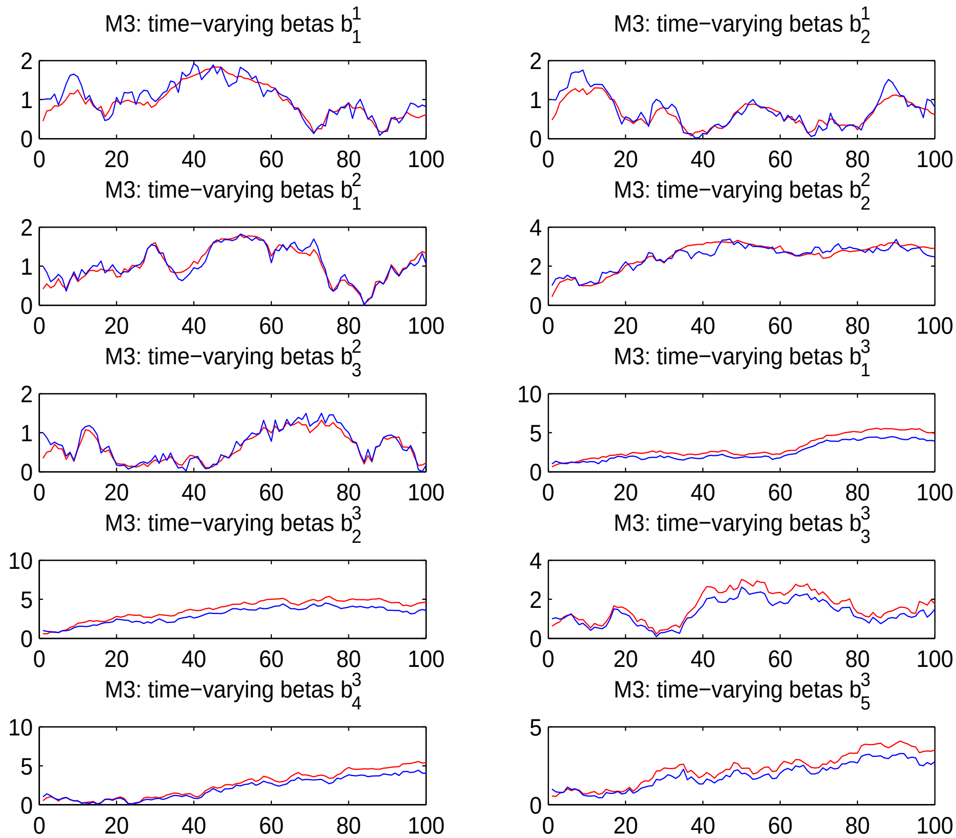

Figure 6: Posterior means for the time-varying parameters $\beta_{i,t}^{j}$, $j = 1, 2, 3$, (red line) and true time-varying beta values (blue line) simulated based on model specification M_3.

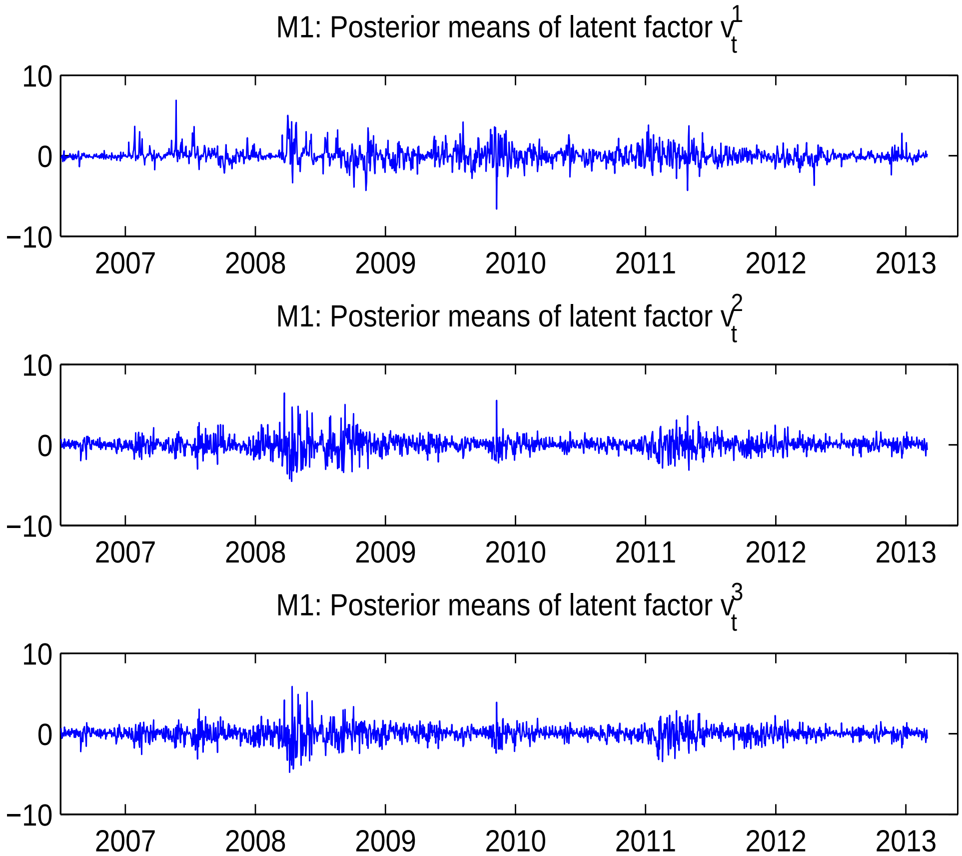

Figure 7: Posterior means for the latent factors v_t^j, $j = 1, 2, 3,$ estimated using model specification M_1.